

# Minimal flavour violation waiting for precise measurements of $\Delta M_s$ , $S_{\psi\phi}$ , $A_{\text{SL}}^s$ , $|V_{ub}|$ , $\gamma$ and $B_{s,d}^0 \rightarrow \mu^+ \mu^-$

Monika Blanke, Andrzej J. Buras, Diego Guadagnoli and Cecilia Tarantino

*Physik Department, Technische Universität München,*

*D-85748 Garching, Germany*

*E-mail: mblanke@ph.tum.de, andrzej.buras@ph.tum.de,*

*diego.guadagnoli@ph.tum.de, cecilia.tarantino@ph.tum.de*

ABSTRACT: We emphasize that the recent measurements of the  $B_s^0 - \bar{B}_s^0$  mass difference  $\Delta M_s$  by the CDF and DØ collaborations offer an important *model independent* test of minimal flavour violation (MFV). The improved measurements of the angle  $\gamma$  in the unitarity triangle and of  $|V_{ub}|$  from tree level decays, combined with future accurate measurements of  $\Delta M_s$ ,  $S_{\psi K_S}$ ,  $S_{\psi\phi}$ ,  $Br(B_{d,s} \rightarrow \mu^+ \mu^-)$ ,  $Br(B \rightarrow X_{d,s} \nu \bar{\nu})$ ,  $Br(K^+ \rightarrow \pi^+ \nu \bar{\nu})$  and  $Br(K_L \rightarrow \pi^0 \nu \bar{\nu})$  and improved values of the relevant non-perturbative parameters, will allow to test the MFV hypothesis in a model independent manner to a high accuracy. In particular, the difference between the *reference* unitarity triangle obtained from tree level processes and the *universal* unitarity triangle (UUT) in MFV models would signal either new flavour violating interactions and/or new local operators that are suppressed in MFV models with low  $\tan \beta$ , with the former best tested through  $S_{\psi\phi}$  and  $K_L \rightarrow \pi^0 \nu \bar{\nu}$ . A brief discussion of non-MFV scenarios is also given. In this context we identify in the recent literature a relative sign error between Standard Model and new physics contributions to  $S_{\psi\phi}$ , that has an impact on the correlation between  $S_{\psi\phi}$  and  $A_{\text{SL}}^s$ . We point out that the ratios  $S_{\psi\phi}/A_{\text{SL}}^s$  and  $\Delta M_s/\Delta\Gamma_s$  will allow to determine  $\Delta M_s/(\Delta M_s)^{\text{SM}}$ . Similar proposals for the determination of  $\Delta M_d/(\Delta M_d)^{\text{SM}}$  are also given.

KEYWORDS: Beyond Standard Model, CP violation, Rare Decays.

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**1. Introduction**

The recent measurement of the  $B_s^0 - \bar{B}_s^0$  mass difference by the CDF collaboration [1]

$$\Delta M_s = (17.33_{-0.21}^{+0.42} \pm 0.07)/\text{ps} \tag{1.1}$$

and the two-sided bound by the DØ collaboration [2]  $17/\text{ps} \leq \Delta M_s \leq 21/\text{ps}$  (90% C.L.) provided still another constraint on the Standard Model (SM) and its extensions. In particular, the value of  $\Delta M_s$  measured by the CDF collaboration turned out to be rather surprisingly below the SM predictions obtained from other constraints [3, 4]

$$(\Delta M_s)_{\text{UTfit}}^{\text{SM}} = (21.5 \pm 2.6)/\text{ps}, \quad (\Delta M_s)_{\text{CKMfitter}}^{\text{SM}} = (21.7_{-4.2}^{+5.9})/\text{ps}. \tag{1.2}$$

The tension between (1.1) and (1.2) is not yet significant, due to the sizable non-perturbative uncertainties. A consistent though slightly smaller value is found for the mass difference directly from its SM expression [5]

$$(\Delta M_s)_{\text{direct}}^{\text{SM}} = \frac{G_F^2}{6\pi^2} \eta_B m_{B_s} \left( \hat{B}_{B_s} F_{B_s}^2 \right) M_W^2 S(x_t) |V_{ts}|^2 = (17.8 \pm 4.8)/\text{ps}, \tag{1.3}$$

with  $S(x_t)$  being the SM Inami-Lim function,  $|V_{ts}| = 0.0409 \pm 0.0009$  and the other input parameters collected in table 1.

It should be emphasized that  $\Delta M_s > (\Delta M_s)^{\text{SM}}$  is favoured in many simple extensions of the SM like Two-Higgs-Doublet Models type II, MSSM with low  $\tan \beta$ , Littlest Higgs

Model without T-Parity [6] and Universal-Extra-Dimensions [7]. A notable exception is the MSSM with minimal flavour violation (MFV) and large  $\tan\beta$ , where the suppression of  $\Delta M_s$  with respect to  $(\Delta M_s)^{\text{SM}}$  has been predicted [8]. In more complicated models, like the MSSM with new flavour violating interactions [9],  $\Delta M_s$  can be smaller or larger than  $(\Delta M_s)^{\text{SM}}$ .

In this paper we would like to emphasize that this new result offers an important *model independent* test of models with MFV [10–12], within the  $B_d^0$  and  $B_s^0$  systems. We will summarize its implications for MFV models and discuss briefly non-MFV scenarios. The first version of our paper appeared few days before the announcement of the result in (1.1) [1], which has considerably reduced the uncertainties and prompted us to extend our analysis.

We will use first a constrained definition of MFV [10], to be called CMFV in what follows, in which

- flavour and CP violation is exclusively governed by the CKM matrix [13]
- the structure of low energy operators is the same as in the SM.

The second condition introduces an additional constraint not present in the general formulation of [11], but has the virtue that CMFV can be tested by means of relations between various observables that are independent of the parameters specific to a given CMFV model [10]. The violation of these relations would indicate the relevance of new low energy operators and/or the presence of new sources of flavour and CP violation, encountered for instance in general supersymmetric models [14]. The first studies of the implications of the  $\Delta M_s$  experimental results on the parameters of such models can be found in [9, 15–19] and the result in (1.1) has been included in the analyses of the UTfit and CKMfitter collaborations [3, 4].

Our paper is organized as follows: section 2 is devoted entirely to CMFV and  $\Delta B = 2$  transitions. In section 3 we study the implications of (1.1) on the CMFV relations between  $\Delta B = 1$  and  $\Delta B = 2$  processes. In section 4 we discuss briefly the tests involving both  $K$  and  $B$  systems. In section 5 we discuss the impact of new operators still in the context of MFV. In section 6 we analyse some aspects of non-MFV scenarios, and in section 7 we have a closer look at the CP asymmetry  $S_{\psi\phi}$  and its correlation with  $A_{\text{SL}}^s$ . In section 8 we give a brief summary of our findings.

## 2. Basic relations and their first tests

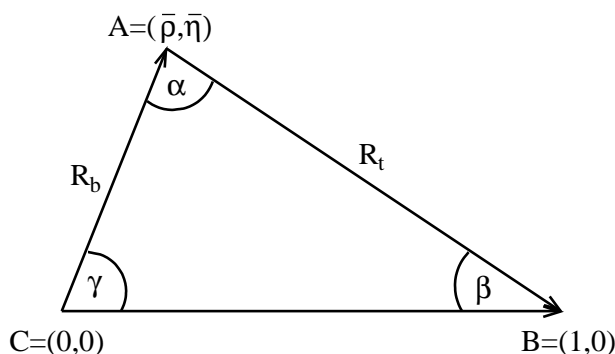
It will be useful to adopt the following sets of fundamental parameters related to the CKM matrix and the unitarity triangle shown in figure 1:

$$|V_{us}| \equiv \lambda, \quad |V_{cb}|, \quad R_b, \quad \gamma, \quad (2.1)$$

$$|V_{us}| \equiv \lambda, \quad |V_{cb}|, \quad R_t, \quad \beta. \quad (2.2)$$

The following known expressions will turn out to be useful in what follows:

$$R_b \equiv \frac{|V_{ud}V_{ub}^*|}{|V_{cd}V_{cb}^*|} = \sqrt{\bar{\varrho}^2 + \bar{\eta}^2} = \left(1 - \frac{\lambda^2}{2}\right) \frac{1}{\lambda} \left| \frac{V_{ub}}{V_{cb}} \right|, \quad (2.3)$$



**Figure 1:** Unitarity Triangle.

$$R_t \equiv \frac{|V_{td}V_{tb}^*|}{|V_{cd}V_{cb}^*|} = \sqrt{(1 - \bar{\rho})^2 + \bar{\eta}^2} = \frac{1}{\lambda} \left| \frac{V_{td}}{V_{cb}} \right|. \quad (2.4)$$

While set (2.1) can be determined entirely from tree level decays and consequently independently of new physics contributions, the variables  $R_t$  and  $\beta$  in set (2.2) can only be determined in one-loop induced processes and are therefore in principle sensitive to new physics. It is the comparison between the values for the two sets of parameters determined in the respective processes, that offers a powerful test of CMFV, when the unitarity of the CKM matrix is imposed. One finds then the relations

$$R_b = \sqrt{1 + R_t^2 - 2R_t \cos \beta}, \quad \cot \gamma = \frac{1 - R_t \cos \beta}{R_t \sin \beta}, \quad (2.5)$$

which are profound within CMFV for the following reasons. The quantities on the l.h.s. of (2.5) can be determined entirely in tree level processes, whereas the variables  $\beta$  and  $R_t$  from one-loop induced processes. The important virtue of CMFV, to be contrasted with other extensions of the SM, is that the determination of  $\beta$  and  $R_t$  does not require the specification of a given CMFV model. In particular, determining  $\beta$  and  $R_t$  by means of

$$\sin 2\beta = S_{\psi K_S}, \quad (2.6)$$

$$\begin{aligned} R_t &= \frac{\xi}{\lambda} \sqrt{\frac{\Delta M_d}{\Delta M_s}} \sqrt{\frac{m_{B_s}}{m_{B_d}}} \left[ 1 - \lambda \xi \sqrt{\frac{\Delta M_d}{\Delta M_s}} \sqrt{\frac{m_{B_s}}{m_{B_d}}} \cos \beta + \frac{\lambda^2}{2} + \mathcal{O}(\lambda^4) \right] \\ &\approx 0.923 \left[ \frac{\xi}{1.23} \right] \sqrt{\frac{17.4/\text{ps}}{\Delta M_s}} \sqrt{\frac{\Delta M_d}{0.507/\text{ps}}}, \end{aligned} \quad (2.7)$$

where [20]

$$\xi = \frac{\sqrt{\hat{B}_{B_s} F_{B_s}}}{\sqrt{\hat{B}_{B_d} F_{B_d}}} = 1.23 \pm 0.06, \quad (2.8)$$

allows to construct the UUT [10] for all CMFV models that can be compared with the reference unitarity triangle [21] following from  $R_b$  and  $\gamma$ . The difference between these two

$G_F = 1.16637 \cdot 10^{-5} \text{ GeV}^{-2}$	$ V_{ub}  = 0.00423(35)$
$M_W = 80.425(38) \text{ GeV}$	$ V_{cb}  = 0.0416(7)$ [23]
$\alpha = 1/127.9$	$\lambda = 0.225(1)$ [24]
$\sin^2 \theta_W = 0.23120(15)$	$F_{B_s} \sqrt{\hat{B}_{B_s}} = 262(35) \text{ MeV}$
$m_\mu = 105.66 \text{ MeV}$	$\xi = 1.23(6)$
$\Delta M_K = 3.483(6) \cdot 10^{-15} \text{ GeV}$	$\hat{B}_{B_d} = 1.28(10)$
$F_K = 159.8(15) \text{ MeV}$	$\hat{B}_{B_s} = 1.30(10)$
$m_{K^0} = 497.65(2) \text{ MeV}$ [22]	$\hat{B}_{B_s}/\hat{B}_{B_d} = 1.02(4)$ [20]
$m_{B_d} = 5.2793(7) \text{ GeV}$	$\eta_1 = 1.32(32)$ [25]
$m_{B_s} = 5.370(2) \text{ GeV}$	$\eta_3 = 0.47(5)$ [26]
$\tau(B_d) = 1.530(9) \text{ ps}$	$\eta_2 = 0.57(1)$
$\tau(B_s) = 1.466(59) \text{ ps}$	$\eta_B = 0.55(1)$ [5]
$\tau(B_s)/\tau(B_d) = 0.958(39)$	$\eta_Y = 1.012(5)$ [27]
$\Delta M_d = 0.507(5)/\text{ps}$	$\bar{m}_c = 1.30(5) \text{ GeV}$
$S_{\psi K_S} = 0.687(32)$ [23]	$\bar{m}_t = 163.8(32) \text{ GeV}$

**Table 1:** Values of the experimental and theoretical quantities used as input parameters.

triangles signals new sources of flavour violation and/or new low energy operators beyond the CMFV scenario. Here,  $S_{\psi K_S}$  stands for the coefficient of  $\sin(\Delta M_d t)$  in the mixing induced CP asymmetry in  $B_d^0(\bar{B}_d^0) \rightarrow \psi K_S$  and, in obtaining the expression (2.7) for  $R_t$ , we have taken into account a small difference between  $|V_{cb}|$  and  $|V_{ts}|$ , that will play a role once the accuracy on  $\xi$  and  $\Delta M_s$  improves.

The values of the input parameters entering in (2.7) and used in the rest of the paper are collected in table 1. In particular, we take as lattice averages of  $B$ -parameters and decay constants the values quoted in [20], which combine unquenched results obtained with different lattice actions.

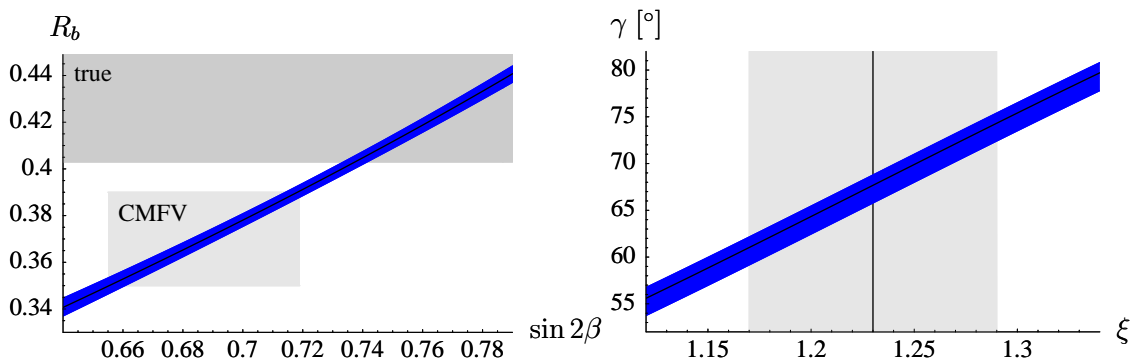
Until the recent measurement of  $\Delta M_s$  in (1.1) [1], none of the relations in (2.5) could be tested in a model independent manner, even if the imposition of other constraints like  $\varepsilon_K$  and separate information on  $\Delta M_d$  and  $\Delta M_s$  implied already interesting results for models with CMFV [3, 4, 28]. In particular in [11] the UUT has been constructed by using  $\varepsilon_K$ ,  $\Delta M_d$  and  $\Delta M_s$  and treating the relevant one-loop function  $S = S(x_t) + \Delta S$  as a free parameter. A similar strategy has been used earlier in [29] to derive a lower bound on  $\sin 2\beta$  from CMFV. While such an approach is clearly legitimate, we think that using only quantities in which one has fully eliminated the dependence on new physics parameters allows a more transparent test of CMFV, and in the case of data indicating departures from CMFV, to identify clearly their origin.

With the measurement of  $\Delta M_s$  in (1.1) at hand,  $S_{\psi K_S}$  and  $\Delta M_d$  known very precisely [23], we find using (2.6) and (2.7)

$$(\sin 2\beta)_{\text{CMFV}} = 0.687 \pm 0.032, \quad (R_t)_{\text{CMFV}} = 0.923 \pm 0.044, \quad (2.9)$$

and subsequently, using (2.5),

$$(R_b)_{\text{CMFV}} = 0.370 \pm 0.020, \quad \gamma_{\text{CMFV}} = (67.4 \pm 6.8)^\circ. \quad (2.10)$$



**Figure 2:**  $R_b$  and  $\gamma$  in CMFV as functions of  $\sin 2\beta$  and  $\xi$ , respectively.

This should be compared with the values for  $R_b$  and  $\gamma$  known from tree level semileptonic  $B$  decays [23] and  $B \rightarrow D^{(*)}K$  [3], respectively

$$(R_b)_{\text{true}} = 0.440 \pm 0.037, \quad \gamma_{\text{true}} = (71 \pm 16)^\circ. \quad (2.11)$$

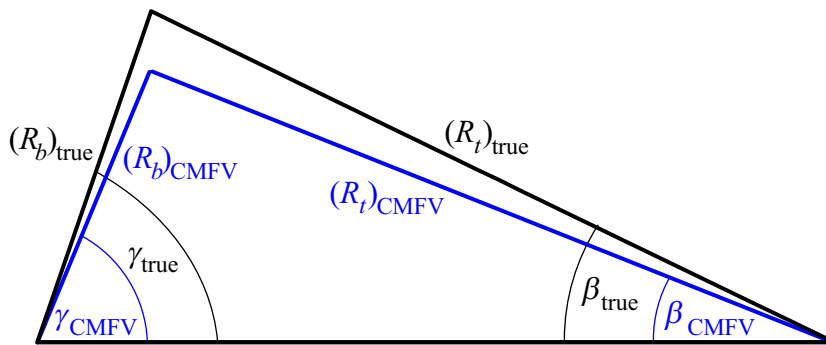
The relations in (2.5) can then be tested for the first time, even if the quality of the test is still not satisfactory. We have dropped in (2.11) the solution  $\gamma = -(109 \pm 16)^\circ$  as it is inconsistent with  $\beta > 0$  within the MFV framework, unless the new physics contributions to the one-loop function  $S$  in  $B_d^0 - \bar{B}_d^0$  mixing reverse its sign [30]. Moreover, it is ruled out by the lower bound on  $\Delta M_s$ .

With future improved measurements of  $\Delta M_s$ , of  $\gamma$  from  $B \rightarrow D^{(*)}K$  and other tree level decays, a more accurate value for  $R_b$  from  $|V_{ub}/V_{cb}|$  and a more accurate value of  $\xi$ , the important tests of CMFV summarized in (2.5) will become effective.

In the left panel of figure 2 we show  $R_b$  as a function of  $\sin 2\beta$  for  $\xi$  and  $\Delta M_s$  varied in the ranges (2.8) and (1.1) respectively. The lower part of the range (2.11) obtained for  $R_b$  from tree level semileptonic decays is also shown. This plot and the comparison of (2.10) and (2.11) show very clearly the tension between the values for  $\sin 2\beta$  and  $R_b$  in (2.9) and (2.11), respectively. We will return to this issue in section 6. For completeness we recall here the even stronger tension that exists between the value of  $R_b$  in (2.11) and the measured  $(\sin 2\beta)_{\phi K_S} = 0.47 \pm 0.19$  [23] coming from the CP asymmetry in  $B_d^0(\bar{B}_d^0) \rightarrow \phi K_S$ , which is sensitive to new physics in the decay amplitude.

In the right panel of figure 2 we show  $\gamma$  as a function of  $\xi$  with  $\Delta M_s$  and  $\sin 2\beta$  varied in the ranges (1.1) and (2.9), respectively. As the uncertainty in this plot originates dominantly from  $\Delta M_s$ , the main impact of the recent measurement of  $\Delta M_s$  in (1.1) is to constrain the angle  $\gamma$  in the UUT. With the sizable errors on  $\xi$  in (2.8) and  $\gamma_{\text{true}}$  in (2.11), the second CMFV relation in (2.5) is satisfied, as seen from (2.10) and (2.11), but clearly this test is not conclusive at present. It will be interesting to monitor the plots in figure 2, when the errors on the values of the quantities involved in these tests will be reduced with time.

Finally, in figure 3 we show the universal unitarity triangle and the reference unitarity triangle, constructed using the central values in (2.9) and (2.11), respectively. The qualitative differences between CMFV and tree determination, to which we will return in section



**Figure 3:** Reference Unitarity Triangle and Universal Unitarity Triangle.

6, can clearly be seen in this figure. However, these differences are small and the basic message of figure 3 is that from the point of view of the so-called “ $B_d$ -triangle” of figure 1, the present measurements exhibit CMFV in a reasonable shape.

### 3. Implications for rare decays

The result for  $\Delta M_s$  in (1.1) has immediately four additional profound consequences for CMFV models:

- The ratio

$$\frac{Br(B_s \rightarrow \mu^+ \mu^-)}{Br(B_d \rightarrow \mu^+ \mu^-)} = \frac{\hat{B}_{B_d} \tau(B_s) \Delta M_s}{\hat{B}_{B_s} \tau(B_d) \Delta M_d} = 32.4 \pm 1.9 \quad (3.1)$$

can be predicted very accurately [31], subject to only small non-perturbative uncertainties in  $\hat{B}_{B_s}/\hat{B}_{B_d}$  and experimental uncertainties in  $\tau(B_s)/\tau(B_d)$ .

- Similarly, one can predict

$$\frac{Br(B \rightarrow X_s \nu \bar{\nu})}{Br(B \rightarrow X_d \nu \bar{\nu})} = \frac{|V_{ts}|^2}{|V_{td}|^2} = \frac{m_{B_d}}{m_{B_s}} \frac{1}{\xi^2} \frac{\Delta M_s}{\Delta M_d} = 22.3 \pm 2.2, \quad (3.2)$$

where the second relation will offer a very good test of CMFV, once  $|V_{ts}|$  and  $|V_{td}|$  will be known from the determination of the reference unitarity triangle and the error on  $\xi$  will be decreased.

- From (3.2) we can also extract

$$\frac{|V_{td}|}{|V_{ts}|} = 0.212 \pm 0.011 \quad (3.3)$$

which, although a bit larger, is still consistent with the results of the UTfit [3] and CKMfitter [4] collaborations and the recent determination of this ratio from  $B \rightarrow V\gamma$  decays [32]:

$$\frac{|V_{td}|}{|V_{ts}|}_{\text{UTfit}} = 0.202 \pm 0.008, \quad \frac{|V_{td}|}{|V_{ts}|}_{\text{CKMfitter}} = 0.2011^{+0.0081}_{-0.0065}, \quad (3.4)$$

$$\frac{|V_{td}|^{\text{Belle}}}{|V_{ts}|_{B \rightarrow V\gamma}} = 0.207 \pm 0.027(\text{exp.}) \pm 0.016(\text{th.}), \quad (3.5)$$

where the values given in (3.4) shifted from  $0.198 \pm 0.010$  and  $0.195 \pm 0.010$ , respectively, due to the inclusion of the recent measurement of  $\Delta M_s$  (1.1) in the analyses.

- The branching ratios for  $B_{s,d} \rightarrow \mu^+ \mu^-$  can be predicted within the SM and any CMFV model with much higher accuracy than it is possible without  $\Delta M_{s,d}$ . In the SM one has [31]

$$Br(B_q \rightarrow \mu^+ \mu^-) = C \frac{\tau(B_q) Y^2(x_t)}{\hat{B}_{B_q} S(x_t)} \Delta M_q, \quad (q = s, d) \quad (3.6)$$

with

$$C = 6\pi \frac{\eta_Y^2}{\eta_B} \left( \frac{\alpha}{4\pi \sin^2 \theta_W} \right)^2 \frac{m_\mu^2}{M_W^2} = 4.39 \cdot 10^{-10} \quad (3.7)$$

and  $S(x_t) = 2.33 \pm 0.07$  and  $Y(x_t) = 0.95 \pm 0.03$  being the relevant top mass dependent one-loop functions.

In figure 4 we plot  $Br(B_d \rightarrow \mu^+ \mu^-)$  and  $Br(B_s \rightarrow \mu^+ \mu^-)$  in the SM as functions of  $\hat{B}_{B_d}$  and  $\hat{B}_{B_s}$ , respectively, with the errors in the other quantities entering (3.6) added in quadrature. Clearly, a reduction of the uncertainties on  $\hat{B}_{B_q}$  is very desirable. For  $Br(B_d \rightarrow \mu^+ \mu^-)$  the updated value obtained by means of (3.6) reads

$$Br(B_d \rightarrow \mu^+ \mu^-)^{\text{SM}} = (1.03 \pm 0.09) \cdot 10^{-10}, \quad (3.8)$$

and with the value for  $\Delta M_s$  in (1.1), we also obtain

$$Br(B_s \rightarrow \mu^+ \mu^-)^{\text{SM}} = (3.35 \pm 0.32) \cdot 10^{-9}. \quad (3.9)$$

These values should be compared with the most recent upper bounds from CDF [33]

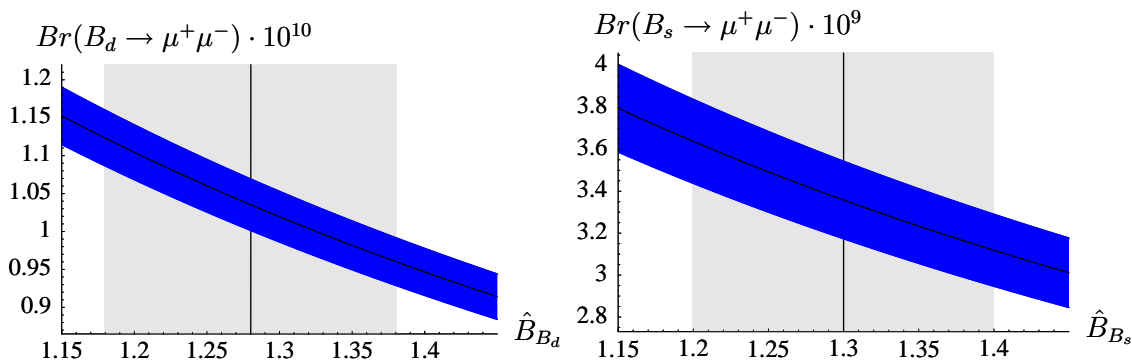
$$Br(B_d \rightarrow \mu^+ \mu^-) < 3 \cdot 10^{-8}, \quad Br(B_s \rightarrow \mu^+ \mu^-) < 1 \cdot 10^{-7} \quad (95\% \text{ C.L.}), \quad (3.10)$$

implying that there is still a lot of room for new physics contributions.

We stress that once LHC is turned on, the accuracy on  $\sin 2\beta$  and  $\Delta M_s$  will match the one of  $\Delta M_d$ , and consequently the accuracy of the predicted values for  $R_b$  and  $\gamma$  in figure 2, of the ratios in (3.1)–(3.3) and of the SM predictions in (3.8) and (3.9) will depend entirely on the accuracy of  $\xi$  and  $\hat{B}_{B_q}$  which therefore has to be improved. The resulting numbers from (3.1)–(3.3) can be considered as “magic numbers of CMFV” and any deviation of future data from these numbers will signal new effects beyond CMFV. We underline the model independent character of these tests.

Another very important test of CMFV and of MFV in general, still within  $B_{s,d}$  decays, will be the measurement of the mixing induced asymmetry in  $B_s^0(\bar{B}_s^0) \rightarrow \psi\phi$  that is predicted within the MFV scenario to be  $S_{\psi\phi} = 0.038 \pm 0.002$  [3, 4]. We will return to this issue in section 7.





**Figure 4:**  $Br(B_d \rightarrow \mu^+\mu^-)$  and  $Br(B_s \rightarrow \mu^+\mu^-)$  in the SM as functions of  $\hat{B}_{B_d}$  and  $\hat{B}_{B_s}$ , respectively.

#### 4. Tests beyond $B_{d,s}$ decays

The tests of CMFV considered so far involve only  $B_d$  and  $B_s$  mesons. Equally important are the tests of the CMFV hypothesis in  $K$  meson decays and even more relevant those involving correlations between  $B$  and  $K$  decays that are implied by CMFV [10].

The cleanest model independent test of MFV in  $K$  decays is offered by  $K \rightarrow \pi\nu\bar{\nu}$  decays, where the measurement of  $Br(K_L \rightarrow \pi^0\nu\bar{\nu})$  and  $Br(K^+ \rightarrow \pi^+\nu\bar{\nu})$  allows a very clean determination of  $\sin 2\beta$  [30, 34] to be compared with the one from  $B_d(\bar{B}_d) \rightarrow \psi K_S$ . The recent NNLO calculation of  $K^+ \rightarrow \pi^+\nu\bar{\nu}$  [35] and the improved calculation of long distance contributions to this decay [36] increased significantly the precision of this test. As the determination of  $\sin 2\beta$  from  $B_d(\bar{B}_d) \rightarrow \psi K_S$  measures the CP-violating phase in  $B_d^0 - \bar{B}_d^0$  mixing, while the one through  $K \rightarrow \pi\nu\bar{\nu}$  measures the corresponding phase in  $Z^0$ -penguin diagrams, it is a very non-trivial MFV test. In fact, similarly to  $S_{\psi\phi}$ , it is a test of the MFV hypothesis and not only of the CMFV one, as due to neutrinos in the final state MFV=CMFV in this case. Unfortunately, due to slow progress in measuring these two branching ratios, such a test will only be possible in the next decade.

Thus, for the time being, the only measured quantity in  $K$  decays that could be used in principle for our purposes is the CP-violating parameter  $\varepsilon_K$ . As it is the only quantity that is available in the  $K^0 - \bar{K}^0$  system, its explicit dependence on possible new physics contributions entering through the one-loop function  $S$  cannot be eliminated within the  $K$  system alone. For this reason the usual analysis of the UUT involved so far only  $|V_{ub}/V_{cb}|$ ,  $S_{\psi K_S}$  and the upper bound on  $\Delta M_d/\Delta M_s$  [3, 37].

Here, we would like to point out that in fact the combination of  $\varepsilon_K$  and  $\Delta M_d$ , used already in [29] to derive a lower bound on  $\sin 2\beta$  from CMFV, can also be used in the construction of the UUT and generally in the tests of CMFV. Indeed, in all CMFV models considered, only the term in  $\varepsilon_K$  involving  $(V_{ts}^*V_{td})^2$  is affected visibly by new physics with the remaining terms described by the SM. Eliminating then the one-loop function  $S$  in  $\varepsilon_K$  in terms of  $\Delta M_d$  one finds following [29]

$$\sin 2\beta = \frac{0.542}{\kappa} \left[ \frac{|\varepsilon_K|}{|V_{cb}|^2 \hat{B}_K} - 4.97\bar{\eta}P_c(\varepsilon_K) \right] \tag{4.1}$$

with

$$\kappa = \left[ \frac{\Delta M_d}{0.507/\text{ps}} \right] \left[ \frac{214 \text{ MeV}}{F_{B_d} \sqrt{\hat{B}_{B_d}}} \right]^2, \quad P_c(\varepsilon_K) = 0.29 \pm 0.06, \quad (4.2)$$

that should be compared with  $\sin 2\beta$  in (2.9). As the second term in (4.1) is roughly by a factor of three smaller than the first term, the small model dependence in  $\bar{\eta}$  can be neglected for practical purposes. The non-perturbative uncertainties in  $\hat{B}_K$  and  $F_{B_d} \sqrt{\hat{B}_{B_d}}$  [20] do not allow a precise test at present, but the situation could improve in the future.

In summary, CMFV has survived its first model independent tests, although there is some tension between the values of  $\beta_{\text{true}}$  and  $\beta_{\text{CMFV}}$ , as seen in figure 3. We will return to this issue in section 6. Due to the significant experimental error in the tree level determinations of  $\gamma$  and  $|V_{ub}/V_{cb}|$  and the theoretical error in  $\xi$ , these tests are not conclusive at present. We are looking forward to the reduction of these errors. This will allow much more stringent tests of CMFV, in particular, if in addition also the tests of model independent CMFV relations discussed above and in [10, 31, 38] that involve rare  $B$  and  $K$  decays will also be available. Future violations of some of these relations would be exciting. Therefore, let us ask next what would be the impact of new operators within MFV on some of the relations discussed above.

## 5. The impact of new operators

In the most general MFV no new phases beyond the CKM one are allowed and consequently (2.6) remains valid. On the other hand in models with two Higgs doublets, like the MSSM, new scalar operators originating dominantly in Higgs penguin diagrams become important at large  $\tan\beta$  and, being sensitive to the external masses, modify  $\Delta M_d$  and  $\Delta M_s$  differently [8]

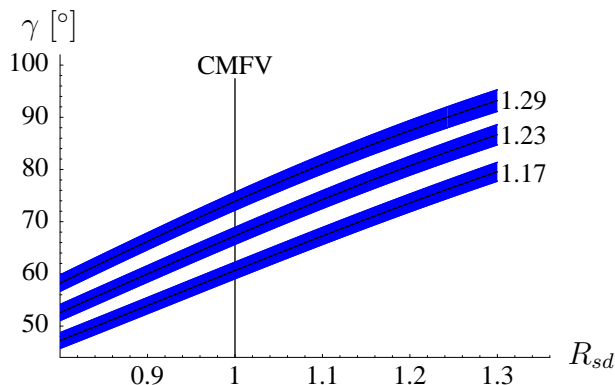
$$\Delta M_q = (\Delta M_q)^{\text{SM}}(1 + f_q), \quad f_q \propto -m_b m_q \tan^2 \beta \quad (q = d, s). \quad (5.1)$$

Consequently the CMFV relation between  $R_t$  and  $\Delta M_d/\Delta M_s$  (2.7) is modified to

$$R_t = 0.923 \left[ \frac{\xi}{1.23} \right] \sqrt{\frac{17.4/\text{ps}}{\Delta M_s}} \sqrt{\frac{\Delta M_d}{0.507/\text{ps}}} \sqrt{R_{\text{sd}}}, \quad R_{\text{sd}} = \frac{1 + f_s}{1 + f_d}. \quad (5.2)$$

In the MSSM at large  $\tan\beta$ ,  $f_s < 0$  and  $f_d \approx 0$  [8], as indicated in (5.1), but as analyzed in [11], more generally  $f_s$  could also be positive. In figure 5 we show the impact of  $R_{\text{sd}} \neq 1$  on the value of  $\gamma$  for different values of  $\xi$  with the errors in the remaining quantities added in quadrature. This figure makes clear that in order to be able to determine  $R_{\text{sd}}$  from the data in this manner, the error in  $\xi$  should be significantly reduced.

The new relation in (5.2) has to be interpreted with some care. After all,  $R_t$  depends only on  $\Delta M_d$  and  $f_d$  and not on  $f_s$  and  $\Delta M_s$ , which has been primarily used in (2.7) and here to reduce the non-perturbative uncertainties due to  $\hat{B}_{B_d} F_{B_d}^2$  in  $\Delta M_d$ . For instance, if  $f_s$  is indeed negative as found in the MSSM with MFV at large  $\tan\beta$ , the measured value of  $\Delta M_s$  will also be smaller cancelling the effect of a negative  $f_s$  in calculating  $R_t$ .



**Figure 5:**  $\gamma$  as a function of  $R_{sd}$  for different values of  $\xi$ .

Thus in the MSSM at large  $\tan\beta$  in which  $f_d \simeq 0$ , the numerical value of  $R_t$  is basically not modified with respect to the SM even if  $\Delta M_s$  measured by CDF appears smaller than  $(\Delta M_s)^{\text{SM}}$  as seen in (1.2).

The fact that  $\Delta M_s$  could indeed be smaller than  $(\Delta M_s)^{\text{SM}}$  is very interesting, as most MFV models studied in the literature, with a notable exception of the MSSM at large  $\tan\beta$  [8], predicted  $\Delta M_s > (\Delta M_s)^{\text{SM}}$ . Unfortunately, finding out whether the experimental value of  $\Delta M_s$  is smaller or larger or equal to  $(\Delta M_s)^{\text{SM}}$  would require a considerable reduction of the uncertainty on  $F_{B_s} \sqrt{\hat{B}_{B_s}}$  that is, at present, roughly 10 – 15%. We will return to this issue in section 7.

In this context let us remark that an improved calculation of  $F_{B_s} \sqrt{\hat{B}_{B_s}}$  together with a rather accurate value of  $|V_{ts}|$  and  $\Delta M_s$  would allow to measure in a model independent manner the function  $S$  and, consequently, to check whether the SM value of this function ( $S(x_t)$  in 1.3) agrees with the experimental one.

Of considerable interest is the correlation between new operator effects in  $\Delta M_s$  and  $Br(B_{s,d} \rightarrow \mu^+ \mu^-)$  that has been pointed out in the MSSM with MFV and large  $\tan\beta$  in [8] and subsequently generalized to arbitrary MFV models in [11]. In particular within the MSSM, the huge enhancement of  $Br(B_{s,d} \rightarrow \mu^+ \mu^-)$  at large  $\tan\beta$  analyzed by many authors in the past [39] is correlated with the suppression of  $\Delta M_s$  with respect to the SM, in contrast to the CMFV relation (3.6). Detailed analyses of this correlation can be found in [8, 40] with the most recent ones in [16, 41, 42]. Here we just want to remark that due to the fact that  $\Delta M_s$  is found close to the SM prediction, no large enhancements of  $Br(B_{d,s} \rightarrow \mu^+ \mu^-)$  are expected within the MSSM with MFV and an observation of  $Br(B_s \rightarrow \mu^+ \mu^-)$  and  $Br(B_d \rightarrow \mu^+ \mu^-)$  with rates  $\text{few} \cdot 10^{-8}$  and  $\text{few} \cdot 10^{-9}$ , respectively, would clearly signal new effects beyond the MFV framework [28]. Indeed such a correlation between  $\Delta M_s$  and  $Br(B_s \rightarrow \mu^+ \mu^-)$  can be avoided in the MSSM with new sources of flavour violation [43].

On the other hand, the fact that  $\Delta M_s$  has been found below its SM expectation keeps the MSSM with MFV and large  $\tan\beta$  alive and this version of MSSM would even be favoured if one could convincingly demonstrate that  $\Delta M_s < (\Delta M_s)^{\text{SM}}$ .

Let us remark that in the case of the dominance of scalar operator contributions to

$Br(B_{d,s} \rightarrow \mu^+\mu^-)$ , the golden relation (3.1) is modified in the MSSM to [31]

$$\frac{Br(B_s \rightarrow \mu^+\mu^-)}{Br(B_d \rightarrow \mu^+\mu^-)} = \frac{\hat{B}_d \tau(B_s) \Delta M_s}{\hat{B}_s \tau(B_d) \Delta M_d} \left[ \frac{m_{B_s}}{m_{B_d}} \right]^4 \frac{1}{1 + f_s} \quad (5.3)$$

with  $f_s$  being a complicated function of supersymmetric parameters. In view of the theoretical cleanness of this relation the measurement of the difference between (3.1) and (5.3) is not out of question. On the other hand, the impact of new operators on relation (4.1) will be difficult to see, as these contributions are small in  $\varepsilon_K$  and  $\Delta M_d$  and the non-perturbative uncertainties involved are still significant.

## 6. A brief look beyond MFV

Finally, let us briefly go beyond MFV and admit new flavour violating interactions, in particular new CP-violating phases as well as  $f_s \neq f_d$ . Extensive model independent numerical studies of the UT in such general scenarios have been already performed for some time, in particular in [3, 4, 44–52], where references to earlier literature can be found. The analysis of [45] has recently been updated in [48] in view of the result in (1.1). Here we want to look instead at these scenarios in the spirit of the rest of our paper.

Let us then first assume as indicated by the plot in figure 2 that indeed the value of  $R_b$  following from (2.5) is smaller than the one following from tree level decays. While in the case of the angle  $\gamma$ , nothing conclusive can be said at present, let us assume that  $\gamma$  found from tree level decays is in the ballpark of  $75^\circ$ , say  $\gamma = (75 \pm 5)^\circ$ , that is larger than roughly  $60^\circ$  found from the UT fits [3, 4]. In fact such large values of  $\gamma$  from tree level decays have been indicated by the analyses of  $B \rightarrow \pi\pi$  and  $B \rightarrow \pi K$  data in [53, 54].

In order to see the implications of such findings in a transparent manner, let us invert (2.5) to find

$$R_t = \sqrt{1 + R_b^2 - 2R_b \cos \gamma}, \quad \cot \beta = \frac{1 - R_b \cos \gamma}{R_b \sin \gamma}. \quad (6.1)$$

In the spirit of the analysis in [54] we then set  $\gamma_{\text{true}} = (75 \pm 5)^\circ$  and  $(R_b)_{\text{true}} = 0.44 \pm 0.04$  and determine the true values of  $\beta$  and  $R_t$ ,

$$\beta_{\text{true}} = (25.6 \pm 2.3)^\circ, \quad (R_t)_{\text{true}} = 0.983 \pm 0.038, \quad (6.2)$$

to be compared with

$$\beta_{\text{CMFV}} = (21.7 \pm 1.3)^\circ, \quad (R_t)_{\text{CMFV}} = 0.923 \pm 0.044, \quad (6.3)$$

that follow from (2.6) and (2.7), respectively. The difference between (6.2) and (6.3) is similar to the one shown in figure 3, though we have chosen here  $\gamma_{\text{true}}$  to be larger than the central value in (2.11). The present data and the assumption about the true value of  $\gamma$  made above then imply that [54]

$$\beta_{\psi K_S} = \beta_{\text{CMFV}} < \beta_{\text{true}}, \quad \sin 2(\beta_{\text{true}} + \varphi_{B_d}) = S_{\psi K_S}, \quad \varphi_{B_d} < 0 \quad (6.4)$$

with  $\varphi_{B_d}$  being a new complex phase, and

$$(R_t)_{\text{CMFV}} < (R_t)_{\text{true}}. \quad (6.5)$$

The result in (6.4) has been first found in [3] but the values of  $R_t$  and  $\gamma$  obtained in [3] are significantly lower than in [54] and here. The pattern in (6.5) has also been indicated by the analysis in [44], but we underline that the possible “discrepancy” in the values of  $\beta$  is certainly better visible than in the case of  $R_t$ .

In particular we find  $\varphi_{B_d} = -(3.9 \pm 2.6)^\circ$  in agreement with [3] and [54]. Note that now  $\sin 2\beta_{\text{true}} = 0.780 \pm 0.051$  in conflict with  $S_{\psi K_S} = 0.687 \pm 0.032$ .

The possibility of a new weak phase in  $B_d^0 - \bar{B}_d^0$  mixing, indicated by (6.4), could be tested in other decays sensitive to this mixing but could more generally also imply new weak phases in other processes. The latter could then be tested through enhanced CP asymmetries,  $S_{\psi\phi}$ ,  $A_{\text{CP}}(B \rightarrow X_s \gamma)$  and  $A_{\text{SL}}^{s,d}$  that are strongly suppressed in MFV models. Such effects could also be clearly seen in  $K_L \rightarrow \pi^0 \nu \bar{\nu}$ .

The origin of a possible disagreement between  $(R_t)_{\text{true}}$  and  $(R_t)_{\text{CMFV}}$  is harder to identify as it could follow from new flavour violating interactions with the same operator structure as in the SM or/and could imply new enhanced operators that are still admitted within the general formulation of MFV [11] as discussed above. Within the  $\Delta F = 2$  processes alone, it will be difficult, if not impossible, to identify which type of violation of CMFV takes place, unless one specifies a concrete model. On the other hand including  $\Delta F = 1$  transitions in the analysis would allow to identify better the origin of the violation of CMFV and MFV relations, but such an analysis is clearly beyond the scope and the spirit of our paper.

## 7. Some aspects of $S_{\psi\phi}$ and $A_{\text{SL}}^s$

In the next years important tests of MFV will come from improved measurements of the time-dependent mixing induced CP asymmetry

$$A_{\text{CP}}^s(\psi\phi, t) = \frac{\Gamma(\bar{B}_s^0(t) \rightarrow \psi\phi) - \Gamma(B_s^0(t) \rightarrow \psi\phi)}{\Gamma(\bar{B}_s^0(t) \rightarrow \psi\phi) + \Gamma(B_s^0(t) \rightarrow \psi\phi)} \equiv S_{\psi\phi} \sin(\Delta M_s t), \quad (7.1)$$

where the CP violation in the decay amplitude is set to zero, and of the semileptonic asymmetry

$$A_{\text{SL}}^s = \frac{\Gamma(\bar{B}_s^0 \rightarrow l^+ X) - \Gamma(B_s^0 \rightarrow l^- X)}{\Gamma(\bar{B}_s^0 \rightarrow l^+ X) + \Gamma(B_s^0 \rightarrow l^- X)} = \text{Im} \left( \frac{\Gamma_{12}^s}{M_{12}^s} \right), \quad (7.2)$$

where  $\Gamma_{12}^s$  represents the absorptive part of the  $B_s^0 - \bar{B}_s^0$  amplitude. The semileptonic asymmetry  $A_{\text{SL}}^s$  has not been measured yet, while its theoretical prediction in the SM has recently improved thanks to advances in lattice studies of  $\Delta B = 2$  four-fermion operators [55] and to the NLO perturbative calculations of the corresponding Wilson coefficients [56, 57].

Both asymmetries are very small in MFV models but can be enhanced even by an order of magnitude if new complex phases are present. This topic has been extensively discussed in the recent literature, in particular in [48] where the correlation between  $A_{\text{SL}}^s$  and  $S_{\psi\phi}$  has been derived and discussed for the first time. Here we would like to point

out that in most recent papers the sign of the new physics contribution to  $S_{\psi\phi}$  is incorrect with an evident consequence on the correlation in question.

Adopting the popular parametrizations of the new physics contributions [3, 47, 48]

$$\Delta M_s \equiv (\Delta M_s)^{\text{SM}} |1 + h_s e^{2i\sigma_s}| \equiv (\Delta M_s)^{\text{SM}} C_{B_s}, \quad (7.3)$$

with

$$1 + h_s e^{2i\sigma_s} \equiv C_{B_s} e^{2i\varphi_{B_s}}, \quad (7.4)$$

we find

$$S_{\psi\phi} = -\eta_{\psi\phi} \sin(2\beta_s + 2\varphi_{B_s}), \quad V_{ts} = -|V_{ts}| e^{-i\beta_s} \quad (7.5)$$

in the parametrization of [3, 47] and

$$S_{\psi\phi} = -\eta_{\psi\phi} \left[ h_s \frac{\sin 2\sigma_s}{C_{B_s}} + \frac{\sin 2\beta_s (1 + h_s \cos 2\sigma_s)}{C_{B_s}} \right] \quad (7.6)$$

in the parametrization of [48] and setting  $\cos 2\beta_s = 1$ , since  $\beta_s \simeq -1^\circ$ . Here  $\eta_{\psi\phi}$  is the CP parity of the  $\psi\phi$  final state, for which we take  $\eta_{\psi\phi} = +1$ . We find then

$$S_{\psi\phi} = \sin(2|\beta_s| - 2\varphi_{B_s}) \approx -\sin 2\varphi_{B_s}, \quad (7.7)$$

or

$$S_{\psi\phi} = -\frac{h_s \sin 2\sigma_s}{C_{B_s}} + \sin 2|\beta_s| \frac{1 + h_s \cos 2\sigma_s}{C_{B_s}} \approx -\frac{h_s \sin 2\sigma_s}{C_{B_s}}. \quad (7.8)$$

While the sign of  $(S_{\psi\phi})^{\text{SM}}$ , obtained from above for  $\sigma_s = 0$ ,  $h_s = 0$ ,  $C_{B_s} = 1$  and  $\varphi_{B_s} = 0$ , agrees with the recent literature, it is important to clarify that the asymmetry  $S_{\psi\phi}$  measures  $\sin(2|\beta_s| - 2\varphi_{B_s})$  and *not*  $\sin(2|\beta_s| + 2\varphi_{B_s})$  as stated in the literature. This is probably not important for the model independent analysis of  $S_{\psi\phi}$  alone, but it is crucial to have correct signs when one works with specific new physics models, where the new phase in  $\Delta B = 2$  observables is generally correlated with the phases in  $\Delta B = 1$  processes, and if different  $\Delta B = 2$  observables are considered simultaneously.

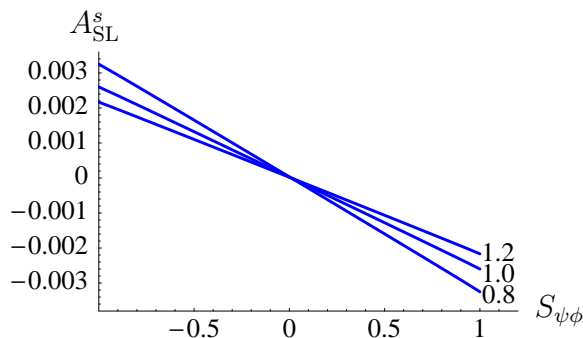
As an example let us consider  $A_{\text{SL}}^s$ , that can be rewritten as

$$\begin{aligned} A_{\text{SL}}^s &= \text{Im} \left( \frac{\Gamma_{12}^s}{M_{12}^s} \right)^{\text{SM}} \frac{\cos 2\varphi_{B_s}}{C_{B_s}} - \text{Re} \left( \frac{\Gamma_{12}^s}{M_{12}^s} \right)^{\text{SM}} \frac{\sin 2\varphi_{B_s}}{C_{B_s}} \\ &\approx -\text{Re} \left( \frac{\Gamma_{12}^s}{M_{12}^s} \right)^{\text{SM}} \frac{\sin 2\varphi_{B_s}}{C_{B_s}}. \end{aligned} \quad (7.9)$$

Recalling that  $\text{Re}(\Gamma_{12}^s/M_{12}^s)^{\text{SM}} < 0$  and using (7.7), we find the following correlation between  $A_{\text{SL}}^s$  and  $S_{\psi\phi}$

$$A_{\text{SL}}^s = - \left| \text{Re} \left( \frac{\Gamma_{12}^s}{M_{12}^s} \right)^{\text{SM}} \right| \frac{1}{C_{B_s}} S_{\psi\phi}, \quad (7.10)$$

shown in figure 6, for different values of  $C_{B_s}$  and with  $|\text{Re}(\Gamma_{12}^s/M_{12}^s)^{\text{SM}}| = (2.6 \pm 1.0) \cdot 10^{-3}$  [56] fixed to its central value. We would like to stress that already a rather small value of  $S_{\psi\phi} \simeq 0.1$  would lead to an order of magnitude enhancement of  $A_{\text{SL}}^s$  relative to its SM expectation.



**Figure 6:**  $A_{\text{SL}}^s$  as a function of  $S_{\psi\phi}$  for different values of  $C_{B_s}$ .

We note that the theoretical prediction for  $\text{Re}(\Gamma_{12}^s/M_{12}^s)^{\text{SM}}$  obtained in [56] and used here is smaller than the value found in [58]. This difference is mainly due to the contribution of  $\mathcal{O}(1/m_b^4)$  in the Heavy Quark Expansion (HQE), which in [58] is wholly estimated in the vacuum saturation approximation (VSA), while in [56] the matrix elements of two dimension-seven operators are expressed in terms of those calculated on the lattice. Moreover, we emphasize that the negative sign in (7.10), now confirmed also in [48], is model independent as  $C_{B_s} = |1 + h_s \exp(2i\sigma_s)| > 0$ . In [48] a first-order expansion in  $h_s$  is performed and the effect of  $C_{B_s}$  is enclosed in neglected  $\mathcal{O}(h_s^2)$  corrections. More generally, for arbitrary  $h_s$  the formula (7.10) is not a simple correlation between  $A_{\text{SL}}^s$  and  $S_{\psi\phi}$ , but a triple correlation between these two quantities and  $C_{B_s}$ . The high generality of this correlation prevents it to be used as a model independent test of New Physics, while in a specific model it can be useful to predict one among these three quantities once the other two are known. Therefore we would like to point out that (7.10) offers in principle an alternative way to find out whether  $\Delta M_s$  differs from  $(\Delta M_s)^{\text{SM}}$ . Indeed, the inversion of (7.10) together with (7.3) yields

$$\frac{\Delta M_s}{(\Delta M_s)^{\text{SM}}} = - \left| \text{Re} \left( \frac{\Gamma_{12}^s}{M_{12}^s} \right)^{\text{SM}} \right| \frac{S_{\psi\phi}}{A_{\text{SL}}^s}. \quad (7.11)$$

With respect to  $(\Delta M_s)^{\text{SM}}$ ,  $\text{Re}(\Gamma_{12}^s/M_{12}^s)^{\text{SM}}$  is free from the uncertainty coming from the decay constant  $F_{B_s}$ . On the other hand, in  $\text{Re}(\Gamma_{12}^s/M_{12}^s)^{\text{SM}}$  significant cancellations occur at NLO and at  $\mathcal{O}(1/m_b^4)$  in the HQE, which make it sensitive to the dimension-seven operators, whose most matrix elements have never been estimated out of the VSA. Future lattice calculations together with experimental measurements of the semileptonic asymmetry  $A_{\text{SL}}^s$  are certainly desired for a significant determination of  $\Delta M_s/(\Delta M_s)^{\text{SM}}$  through (7.11).

Similarly, one has in the  $B_d$  system

$$\frac{\Delta M_d}{(\Delta M_d)^{\text{SM}}} = \left| \text{Re} \left( \frac{\Gamma_{12}^d}{M_{12}^d} \right)^{\text{SM}} \right| \frac{\sin 2\varphi_{B_d}}{A_{\text{SL}}^d} + \text{Im} \left( \frac{\Gamma_{12}^d}{M_{12}^d} \right)^{\text{SM}} \frac{\cos 2\varphi_{B_d}}{A_{\text{SL}}^d}, \quad (7.12)$$

where  $\varphi_{B_d}$  is the new phase in (6.4). We note that in this case  $\text{Im}(\Gamma_{12}^d/M_{12}^d)^{\text{SM}} = -(6.4 \pm 1.4) \cdot 10^{-4}$  cannot be neglected with respect to  $|\text{Re}(\Gamma_{12}^d/M_{12}^d)^{\text{SM}}| = (3.0 \pm 1.0) \cdot 10^{-3}$  [56].

Finally, one could use

$$\frac{\Delta M_q}{(\Delta M_q)^{\text{SM}}} = - \left( \frac{\Delta M_q}{\Delta \Gamma_q} \right) \text{Re} \left( \frac{\Gamma_{12}^q}{M_{12}^q} \right)^{\text{SM}} \cos 2\varphi_{B_q}, \quad (7.13)$$

with  $\varphi_{B_q}$  extracted from  $S_{\psi\phi}$  and  $S_{\psi K_S}$  for  $q = s$  and  $q = d$ , respectively. These proposals have been recently adopted in [59] where an extensive phenomenological analysis in the Littlest Higgs Model with T-parity has been performed. It remains to be seen whether in the future our proposals to measure the ratios  $\Delta M_q/(\Delta M_q)^{\text{SM}}$  by means of (7.11)–(7.13) will be more effective than the direct calculations of  $(\Delta M_q)^{\text{SM}}$ .

## 8. Conclusions

The recent measurements of  $\Delta M_s$  by the CDF and DØ collaborations gave another support to the hypothesis of MFV. Even if possible signals of non-MFV interactions, like  $\varphi_{B_d} \neq 0$  and  $(R_t)_{\text{CMFV}} < (R_t)_{\text{true}}$ , are indicated by the data, they are small as seen in figure 3. However, it should be emphasized that future measurements of CP violation in  $B_s$  decays, in particular of the CP asymmetries  $S_{\psi\phi}$  and  $A_{\text{SL}}^s$  and of the branching ratios  $Br(B_{d,s} \rightarrow \mu^+\mu^-)$ , could modify our picture of non-MFV effects significantly. Also the signals of new weak phases in  $B \rightarrow \pi K$  decays, discussed in [54] and references therein, should not be forgotten.

In the present paper we have concentrated on quantities like ratios of branching ratios,  $\Delta M_d/\Delta M_s$  and various CP asymmetries which do not require the direct use of the weak decay constants  $F_{B_q}$  that are plagued by large non-perturbative uncertainties. Observables sensitive only to  $\xi$  and  $\hat{B}_{B_q}$  have a better chance to help us in identifying new physics contributions. One of the important tasks for the coming years will be to find out whether the data favour positive or negative new physics contributions to  $\Delta M_q$ . As seen in (1.3), from the present perspective, this will not be soon possible through a direct calculation of  $\Delta M_q$ . Therefore, we have proposed the formulae (7.11)–(7.13) as alternative ways to shed light on this important question. We are aware that also these routes are very challenging but they definitely should be followed once the data on  $A_{\text{SL}}^q$  and improved data on  $\Delta \Gamma_q$  will be available.

Truly exciting times are coming for MFV. We should be able to decide in about 2 – 3 years, whether this simple hypothesis survived all model independent tests summarized in this paper, with the final precise tests of the correlations between  $B$  and  $K$  systems left for  $K \rightarrow \pi\nu\bar{\nu}$  in the first years of the next decade. On the other hand if non-MFV interactions will be signalled by the data, flavour physics will be even more exciting. We hope that the formulae and plots collected above will help in monitoring these events in a transparent manner.

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